ON THE STATISTICAL MECHANICS OF PARTICLES SUSPENDED IN A GAS STREAM

PMM Vol. 32, No. 1, 1968, pp. 95-105

Iu.A. BUEVICH
(Moscow)

(*Received June 22, 1967*)

A statistical theory of phase local pulsating motions in uniform flows of a two-component monodisperse system of gas and particles is presented here on the assumption that variations of the mean flow induced by pulsations are comparatively small. It is shown that variation of pulsation velocities of individual particles cannot be considered as a Markovian process, hence the motion of an aggregate of particles in a stream of gas cannot generally be presented as a random process with independent increments. Expressions defining the random motion intensity, and the fluctuating streams of phases are derived together with the transport factors of the suspended particles system.

1. Pattern of random motions. In the following we shall consider stationary uniform flows of the dispersed medium only in which the macroscopic variables (such as, volume concentration of particles ρ, velocities $\mathbf{v}$ and $\mathbf{w}$ of fluid and the dispersed phase in a hydrodynamic approximation, etc.) are independent of coordinates and time. The dispersed phase may in its zero approximation be considered as an ordered lattice of particles in a stream of gas, provided there is no interaction between particles, and no changes of their relative position take place. Equations presented in papers [1 to 3] in which the dispersed phase was considered as a perfect continuous medium are valid in this approximation. Assuming for simplicity's sake the ordinary viscous interaction force between the phases to be linearly dependent on the interphase slip, and neglecting the weight of gas, we obtain in a system of coordinates where $\mathbf{w}_0 = 0$ and $\mathbf{v}_0 = (u, 0, 0)$, the following Eqs.: (1.1)

$$-(1-\rho)\frac{dp}{dx} - \beta\rho K u = 0, \qquad -\rho\frac{dp}{dx} + \beta\rho K u - d_2 g\rho = 0, \qquad \beta = \frac{9}{2}\frac{\mu_0}{a^2}$$

Here p and μ_0 are respectively the gas pressure and viscosity, d_2 the density of the particle material, a the particle radius, g the free fall acceleration and $K(\rho)$ a function which takes into account the increase of the effective viscous resistance force of the ordered undeformable lattice of particles in a constrained flow pattern ($K(0) = 1$, $dK/d\rho > 0$).

The solution of Eqs. (1.1) is of the form

$$p = \text{const} - d_2 g\rho x, \qquad u = (1-\rho)\, d_2 g\, (\beta K)^{-1}, \qquad \mathbf{u} = -u\,(\mathbf{g}\,/\,g) \qquad (1.2)$$

Actually each of the particles suspended in the stream is subject to random pulsations which lead to oscillations of the lattice instantaneous nodes and local disturbances of its order. Such fluctuations of a particle result in the appearance of local perturbations in the flow in the vicinity of that particle, substantially affecting the flow past particles in its immediate neighborhood. These individual small scale motions according to the pattern of paper [4] lead to porosity fluctuations of the dispersed medium, and disturb the balance of forces expressed by Eqs. (1.1). As the result of this there appear in the system macroscopic fluctuations of the gas stream velocity and pressure on a scale considerably greater than the average distance between particles. These fluctuations lead to large scale, essentially anisotropic pulsations of large clusters ('packets') of particles. The physical aspects of this model, and its correlation with experimental data were considered in great detail in [4]. Its latest experimental confirmation may be found, for example, in [5] where fluidization by air of fairly large hollow spheres was investigated, and which succeeded in drawing a clear distinction between the motion of packets and the pulsation of individual particles within these, resulting in their dispersal.

A particle instantaneous velocity may be represented as the sum of $\delta \mathbf{w} + \mathbf{W}$, where $\delta \mathbf{w}$ is the dispersed phase hydrodynamic velocity averaged over a very large number of particles (packet velocity) and $\mathbf{W}$ the velocity of the particle individual motion within the packet For simplicity's sake we shall assume that velocities $\delta \mathbf{w}$, $\mathbf{W}$, as well as the gas mean velocity fluctuation $\delta \mathbf{v}$ are small in comparison with $\mathbf{u}$. With this assumption and the use of relationships (1.2) which characterize the behavior of the ordered lattice of noninteracting particles, we can consider the linearized problem as the zero approximation.

In the stream each particle is acted upon by a force which may be expressed as the sum of orderly and sporadic components. The orderly force calculated per unit volume of particles is given by the second of Eqs. (1.1). The sporadic force may also be resolved into two components the first of which is associated with variations of the orderly force at macroscopic fluctuations of parameters, and is of the form

$$-v^\circ \frac{\partial \delta p}{\partial x_i} + \alpha_0 K(\delta v_i - \delta w_i) + \alpha_0 \frac{dK}{d\rho} u \delta_{1i} \delta\rho, \qquad \alpha_0 = \beta v^\circ \tag{1.3}$$

Here v° is the volume of a particle. The second component is primarily the result of the viscous resistance due to the small scale motion and, secondly, to particles random interaction with local perturbations of the carrier stream which are of a magnitude of the order of the mean distance between particles, and also to direct collisions with neighboring particles. Summarizing the results, we obtain for the particle the Langevin Eq. of the form

$$m \frac{d}{dt}(\delta w_i + W_i) = -v^\circ \frac{\partial \delta p}{\partial x_i} + \alpha_0 K(\delta v_i - \delta w_i) + \alpha_0 \frac{dK}{d\rho} u \delta_{1i} \delta\rho - \alpha W_i + F_i \tag{1.4}$$

Here $m = d_2 v^\circ$ is the particle mass, $\mathbf{F}$ a random force, while coefficient α differs from α_0 in (1.3) in that only that in it the effective viscosity μ, as computed in [6], has been substituted for the physical gas viscosity μ_0. For $\delta v_i = \delta w_i = \delta p = \delta\rho = 0$ Eq. (1.4) becomes the Langevin equation for a Brownian particle [7] in a medium of viscosity μ.

Summating (1.4) over a large number of particles in a unit of volume (by a suitable selection of this unit n may be always made large), we obtain the equation of motion of the dispersed phase in the hydrodynamic approximation

$$d_2 \rho \frac{\partial \delta w_i}{\partial t} = -\rho \frac{\partial \delta p}{\partial x_i} + \beta\rho K(\delta v_i - \delta w_i) + \beta\rho \frac{dK}{d\rho} u \delta_{1i} \delta\rho \tag{1.5}$$

It has been taken into account here that for large n we have $\Sigma W_i \sim n^{1/2} \langle |W_i| \rangle$ and $\Sigma F_i \sim n^{1/2} \langle |F_i| \rangle$, while the Lagrangian derivative appearing in (1.4) has been replaced by an Eulerian one, which is justified by virtue of the assumed smallness of $\delta \mathbf{w} + \mathbf{W}$. Eq. (1.5) coincides with the linearized equation which can be obtained from the dispersed phase equation of motion [1 to 3], if it is assumed that in this phase the momentum transfer is accomplished by its motion only.

We shall use for expressing velocity $\delta \mathbf{v}$ the linearized equation derived from the equation of motion of the dispersion medium [1 to 3]. Taking into account (1.2) and neglecting the gas inertia and viscous energy dissipation, we obtain

$$0 = -(1-\rho) \frac{\partial \delta p}{\partial x_i} - \beta\rho K(\delta v_i - \delta w_i) - \beta\rho \frac{dK}{d\rho} u \delta_{1i} \delta\rho - d_2 g \delta_{1i} \delta\rho \tag{1.6}$$

For an incompressible gas the mass conservation equation becomes after linearization

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}\right) \delta\rho = (1-\rho) \frac{\partial \delta v_i}{\partial x_i} \tag{1.7}$$

Small scale motions of particles tend to attenuate fluctuations of the dispersed phase porosity. We describe this process by means of the diffusion Eq.

$$\frac{\partial \delta\rho}{\partial t} = D \frac{\partial^2 \delta\rho}{\partial x_i \partial x_i} \tag{1.8}$$

Here D is the coefficient of self-diffusion of particles resulting from scale isotropic motions.

We note that fluctuations of the effective pressure in (1.5) and (1.6) are associated with the momentum transfer not only by the thermal motions of molecules, but also by local perturbations which of course, are not described by the averaged Eqs. (1.6). Hence, in the general case δp is a tensor. In view of our problem symmetry only the diagonal components of this tensor differ from zero, and $\delta p_1 \neq \delta p_2 = \delta p_3$.

2. Stochastic equations and their solution. In the following we shall make use of the apparatus of the correlation theory of stationary random fluctuation processes. We shall represent random magnitudes in the form of Fourier-Stiltjes stochastic integrals

$$\{\delta \mathbf{v}, \delta \mathbf{w}, \delta p, \delta\rho\} = \int e^{i\omega t + i\mathbf{kr}} \{d\mathbf{Z}_v, d\mathbf{Z}_w, dZ_p, dZ_\rho\} \tag{2.1}$$

Here integration is carried out over the whole wave space and all frequencies, assuming that random processes dZ in (2.1) satisfy all necessary requirements, and represent differentials of random functions of point $(\omega, \mathbf{k})$ with uncorrelated increments [8]. The spectral densities of various correlations are defined as the second moments of the corresponding magnitudes of dZ; for example, the spectral density of the space-time correlation of magnitude $\delta\rho$ is defined by the equality

$$f_{\rho\rho}(\omega, \mathbf{k}) = \lim_{d\mathbf{k},\, d\omega \to 0} \frac{\langle dZ_\rho(\omega, \mathbf{k})\, dZ_\rho^*(\omega, \mathbf{k})\rangle}{dk_1 dk_2 dk_3 d\omega}$$

In the following we shall separate the isotropic part of all random processes

$$\delta v_i = v'\delta_{i1} + v_i'', \quad \delta w_i = w'\delta_{i1} + w_i'', \quad \frac{\partial \delta p}{\partial x_i} = \frac{\partial p'}{\partial x}\delta_{1i} + \frac{\partial p''}{\partial x_i} \tag{2.2}$$

$$dZ_{vi} = dZ_v'\delta_{1i} + dZ_{vi}'', \quad dZ_{wi} = dZ_w'\delta_{1i} + dZ_{wi}'', \quad k_i dZ_p = k_1 dZ_p' + k_i dZ_p''$$

Substituting (2.1) and (2.2) into Eqs. (1.5) to (1.7), and separating in the latter their isotropic and anisotropic parts, we obtain two systems of Eqs.

$$-i\rho k_1 dZ_p' + \beta\rho K(dZ_v' - dZ_w') + \beta\rho \frac{dK}{d\rho} u dZ_\rho = i d_2 \rho\omega dZ_w'$$

$$-i(1-\rho)k_1 dZ_p' - \beta\rho K(dZ_v' - dZ_w') - \beta\rho \frac{dK}{d\rho} u dZ_\rho - d_2 g dZ_\rho = 0$$

$$(1-\rho)k_1 dZ_v' = u k_1 dZ_\rho \tag{2.3}$$

$$-i\rho k_i dZ_p'' + \beta\rho K(dZ_{vi}'' - dZ_{wi}'') = i d_2 \rho\omega dZ_{wi}''$$

$$-i(1-\rho)k_i dZ_p'' - \beta\rho K(dZ_{vi}'' - dZ_{wi}'') = 0, \qquad (1-\rho)k_i dZ_{vi}'' = \omega dZ_\rho \tag{2.4}$$

Solutions of Eqs. (2.3) and (2.4) are of the form

$$dZ_w' = \frac{\beta K u}{\beta K + i d_2(1-\rho)\omega}\left(\frac{2}{1-\rho} + \frac{d \ln K}{d\rho}\right) dZ_\rho, \quad dZ_v' = \frac{u}{1-\rho} dZ_\rho \tag{2.5}$$

$$dZ_p' = d_2 k_1^{-1}(-\rho\omega dZ_w' + i g dZ_\rho)$$

$$dZ_w'' = \frac{\beta K}{\beta K + id_2(1-\rho)\omega}\,\frac{\omega\mathbf{k}}{(1-\rho)k^2}\,dZ_\rho, \quad dZ''_v = \frac{\omega\mathbf{k}}{(1-\rho)k^2}\,dZ_\rho$$

$$dZ_p'' = -d_2\rho\omega k^{-2}(\mathbf{k}dZ_w'') \tag{2.6}$$

These equations make it possible to find all of the unknown spectral densities, when the spectral density of process $\delta\rho$ is known. The latter must be, of course, determined from independent considerations.

We note that the problem here considered is a natural generalization of the problem of fluid filtration through a random porosity medium which was investigated in [9]. The large scale motions in a disperse system may, by analogy with filtration, be called pseudoturbulence, an important difference being that in the case of pseudoturbulence in a disperse system, contrary to that of a filtration system, the dispersed phase which simulates a porous medium is itself drawn into these pseudoturbulent motions.

3. Spectral density of the random process $\delta\rho$. Parameter δn which defines the divergence of the actual number of particles in a unit of volume from its mean value may in the case of point particles be expressed as the sum of δ-functions defining the positions of individual particles in space [10]. In view of the equivalence of particles and of the statistical uniformity of the space, values of δn in different volumes at any fixed instant of time can be considered as independent, i.e. the spectral density of the δn-process defined with respect to simultaneous two-point correlations is independent of $\mathbf{k}$ [8]. This obviously also holds for the $\delta\rho$-process. Hence, using the diffusion Eq. (1.8) for the spectral density $f_{\rho\rho}(\omega, \mathbf{k})$, we obtain relationship

$$f_{\rho\rho}(\omega, \mathbf{k}) = C'Dk^2(\omega^2 + D^2k^4)^{-1}, \qquad C' = \text{const} \tag{3.1}$$

Actually particles occupy a finite volume, and their positions are determined not by the δ-functions, but within the accuracy of function

$$\Theta(\mathbf{r} - \mathbf{r}_j(t)) = (v^\circ/\rho)^{-1}\,Y(b - |\mathbf{r} - \mathbf{r}_j(t)|), \qquad b = a\rho^{-1/3}$$

where Y is the Heaviside function, and the introduction of function $\Theta(\mathbf{r})$ conforms to the procedure of smoothing the spectrum short wave details as suggested by Massignion [10], while the corresponding spectral density of the $\delta\rho$-process differs from that of (3.1) which characterizes a system of point particles with coordinates defined in detail by factor

$$F\Theta(\mathbf{k}) = \frac{\rho}{v^\circ}\int e^{i\mathbf{kr}}Y(b - |\mathbf{r}|)\,d\mathbf{r} = 3\,\frac{\sin kb - kb\cos kb}{k^3b^3} \tag{3.2}$$

Hence, instead of (3.1) we obtain the following Expression:

$$f_{\rho\rho}(\omega, k) = \frac{CDk^2}{\omega^2 + D^2k^4}\,\frac{\sin kb - kb\cos kb}{(kb)^3}, \qquad C = 3C' \tag{3.3}$$

We note that the condition of equality of the total number of degrees of freedom of the system of particles in the wave space and the number actual degrees of freedom of all particles was used in [4] instead of the smoothing function $\Theta(\mathbf{r})$. In that case integration over the wave space is in fact replaced by summation over the Brillouin zone, i.e. in short wave spectrum area truncation is substituted for the multiplication of function (3.1) by (3.2). The disadvantage of this method based on Debye's ideas is that it does not allow an unambiguous selection of the required number of harmonics in the wave space, and thus contains an element of arbitrariness in the determination of the importance of various harmonics. It is clear that the true importance is defined by function (3.3).

For the computation of constant C in (3.3) we shall determine independently the fluctuation $\langle\delta n^2\rangle$. We introduce the numbers of 'cells' N_V and N contained in volumes V and $A = 1$, with $V \gg 1$, which correspond to the number of particles which can be tightly packed in these volumes. Obviously

$$N_V = V(\rho_* / v^\circ), \qquad N = \rho_* / v^\circ$$

where ρ^* is the volumetric concentration, and v°/ρ_* the specific volume of a particle in a tightly packed system. Let the number of particles in volume V, i.e. the number of occupied cells, be $n_V \leqslant N_V$. The probability of the presence of n particles in volume A is to be determined.

Assuming that all cells of the lattice simulating volume V are equivalent, and examining the process of filling an empty lattice of N_V cells with n_V particles, we obtain for the sought probability the expression

$$P_V(n) = \binom{N_V}{n_V}^{-1} \binom{N_V - N}{n_V - n} \binom{N}{n}$$

It will be readily seen that this distribution satisfies the completeness condition. The limit form of function $P_V(n)$ for $V, n_V \to \infty$, but $\nu = n_V/N_V$ = const, is of interest. With the use of Stirling formula we obtain

$$P(n) = \lim_{V\to\infty} P_V(n) = \binom{N}{n} \nu^n (1-\nu)^{N-n}, \qquad \nu = \lim_{V\to\infty}\left(\frac{n_V}{N_V}\right) = \frac{\rho}{\rho_*} \tag{3.4}$$

The form of this distribution is similar to that of the analogous distribution of diluted colloidal systems [7], but its parameters have an entirely different meaning. In particular for finite values of ν, the Poisson distribution obtained from (3.4) in the limit $\nu \to 0$ widely used in physics of diluted systems, has no meaning in the case of the system here considered. The expressions for moments follow from (3.4)

$$\langle n\rangle = \nu N, \quad \langle n^2\rangle = \nu N(\nu N + 1 - \nu), \quad \langle \delta n^2\rangle = \nu N \quad (1-\nu) \approx n(1-\nu) \tag{3.5}$$

We may note that it is not difficult to derive an explicit expression for the temporal correlation of magnitude δn by using the known method of Smolukhovskii [7]. It is clear Expressions (3.5) obtained from the discrete model yield adequate results for the definition of particle fluctuations in continuous volumes at the limit $v^\circ(\rho A)^{-1} \to 0$.

It follows on the other hand from (3.3) that

$$\langle \delta n^2\rangle = \frac{1}{(v^\circ)^2}\left\langle \left| \int_A \delta\rho d\mathbf{r} \right|^2 \right\rangle = \frac{8C}{(v^\circ)^2} \int d\omega \int d\mathbf{k} \prod_{j=1}^{3} \left(\frac{1-\cos k_j l_j}{k_j^2}\right) \times$$

$$\times f_{\rho\rho}(\omega, \mathbf{k}) = \frac{8\pi^4 C l_1 l_2 l_3}{3(v^\circ)^2}, \qquad l_1 l_2 l_3 = A = 1 \tag{3.6}$$

In accordance with previous statements it was assumed when carrying out integration that $a \to 0$, hence Expression (3.2) is equal to unity. Comparing (3.5) and (3.6) we obtain the formula for C,

$$C = \frac{3n(1-\nu)(v^\circ)^2}{8\pi^4} = \frac{3v^\circ}{8\pi^4} \frac{\rho(\rho_* - \rho)}{\rho_*} \tag{3.7}$$

It will be seen from (3.3), as well as from Expressions (2.5), (2.6) that the temporal scale of macroscopic correlations is defined by the self-diffusion coefficient D, magnitude β and other phase parameters, with the scale of force variation (1.3) appearing in the Langevin Eq. (1.4) of the same order of magnitude as that of the time of significant changes of velocity $\delta\mathbf{w}$. It is thus clear that neither the packet motion, nor the complete motion of individual particles can be considered as a random process with independent increments, irrespective of statistical properties of the random force $\mathbf{F}$ in (1.4). In particular, the type of Fokker-Planck, or Smolukhovskii equations previously widely used in this analysis [11 and 12] do not hold for the statistical analysis of suspended particles systems.

4. Random motion intensity and fluctuating phase streams. Expressions (3.3), (3.7) make the computation of complete space-time correlations of all of the (2.5) and (2.6) processes possible. We shall consider here only certain simultaneous single-point correlations. From (2.5) and (2.6) we obtain the following equalities:

$$\langle \delta w_i \delta w_j\rangle = \int d\omega \int d\mathbf{k} f_{w_i w_j}(\omega, \mathbf{k}) = 0 \qquad (i \neq j)$$

A similar statement is also true for the random process $\delta\mathbf{v}$; it follows as well from the motion symmetry relative to the planes of the carrier stream flow direction.

The computation of spectral density by means of (2.5) and (2.6) and integration with respect to ω and $\mathbf{k}$ yields for the mean-square velocities of the random fluctuating motion of particles of a packet the following relationships:

$$\langle \delta w_1^2 \rangle = \langle w'^2 \rangle + \langle w_1''^2 \rangle, \qquad \langle \delta w_2^2 \rangle = \langle \delta w_3^2 \rangle = \langle w_1''^2 \rangle$$

$$\langle w'^2 \rangle = \frac{\rho^2(\rho_* - \rho)}{\rho_*}\left(\frac{2}{1-\rho} + \frac{d \ln K}{d\rho}\right)^2 \left[1 - \left(1 + \frac{a}{h}\right)\exp\left(-\frac{a}{h}\right)\right] u^2 \qquad (4.1)$$

$$\langle w_1''^2 \rangle = \frac{1}{3}\,\frac{\rho^2(\rho_* - \rho)}{\rho_*}\,\frac{K}{(1-\rho)^3}\left[1 - \left(1 + \frac{a}{h}\right)\exp\left(-\frac{a}{h}\right)\right]\frac{\beta D}{d_2}$$

$$h = \rho^{1/3}\left[\frac{d_2(1-\rho)D}{\beta K}\right]^{1/2} = \rho^{1/3}(1-\rho)^{1/2}\left(\frac{2}{9}\,\frac{D}{K\nu_0}\right)^{1/2} a \qquad \left(\nu_0 = \frac{\mu_0}{d_2}\right)$$

It will be seen from this that the mean velocity of packet motions in the stream flow direction may considerably exceed the mean velocity in the direction normal to it. Expressions for the mean-square velocities of the dispersion medium are easily derived in exactly the same manner from (2.5) and (2.6). We should note, however, that the integral of $f_{v_i v_i}(\omega, \mathbf{k})$ with respect to ω is divergent. This difficulty stems from the use of the diffusion Eq. (1.8), and is easily overcome by the substitution for (1.8) of the more accurate hyperbolic type equation which takes into account the finite velocities of particle translations in a diffusion process.

Actual volume flows of phases are represented in the form of sums of flows corresponding to the ordered lattice model and of additional fluctuating streams associated with large scale pulsations of phases. Hence, in a coordinate system in which $\mathbf{w}_0 \equiv 0$ we have

$$Q_v = (1-\rho)u + Q_v', \qquad Q_v' = -\langle \delta\rho\delta v_1 \rangle, \qquad Q_w = Q_w' = \langle \delta\rho\delta w_1 \rangle$$

Computations yield the following expressions for the fluctuating streams:

$$Q_v' = -\frac{\rho^2(\rho_* - \rho)}{\rho_*}\,\frac{u}{1-\rho} < 0 \qquad (4.2)$$

$$Q_w' = \frac{\rho^2(\rho_* - \rho)}{\rho_*}\left(\frac{2}{1-\rho} + \frac{d \ln K}{d\rho}\right)\left[1 - \left(1 + \frac{a}{h}\right)\exp\left(-\frac{a}{h}\right)\right] u > 0$$

We note that by virtue of the problem symmetry the fluctuating streams of the phases are identically zero in directions normal to that of the carrying stream.

The mean volume velocities of the phases do not, therefore, coincide with the velocities defined by (1.2), and in the system here considered are

$$v_1 = u + (1-\rho)^{-1} Q_v', \qquad w_1 = \rho^{-1} Q_w'$$

For a given concentration the effective mean volume velocity of the interphase slip is

$$u_* = v_1 - w_1 = u + (1-\rho)^{-1} Q_v' - \rho^{-1} Q_w' < u$$

which may be considered as due to the increased viscous resistance of an actual lattice of pulsating particles as compared to the resistance of the ordered lattice. We note that a similar effect is also characteristic of fluid filtration through a medium of random porosity [9].

It is not difficult to compute with the aid of known space-time correlations the scales of various pulsations in different directions, as well as the time scales, and then with the use of known formulas [13] to find transport factors associated with large scale motions. For example, for congruent pseudoturbulent viscosity tensors and for the dispersed phase diffusion we have in the approximation here considered [13]

$$\zeta_{ij} = 0, \quad (i \neq j) \qquad \zeta_{ii} = \zeta_i(\tau) = 2\langle \delta w_i^2 \rangle \int_0^{\tau} (\tau - s) R_{ii}(s)\, ds$$

where $R_{ii}(\tau)$ is the Lagrangian time correlation coefficient of the δw_i-process which for small $\delta \mathbf{w}$ coincides with the Eulerian time correlation coefficient.

5. Small scale motions of particles. From (1.4) and (1.5) we derive the Langevin equation for the small scale motions of a sample particle

$$m\frac{d\mathbf{W}}{dt} = -\alpha\mathbf{W} + \mathbf{F} \tag{5.1}$$

which is similar in its form to the Langevin equation of a Brownian particle.

The random force $\mathbf{F}$ appearing in (5.1) may be conditionally resolved into two components $\mathbf{F}_1$ and $\mathbf{F}_2$, the first of which is related to the interaction of particles and local perturbations of the carrying stream, which in essence is the permanent interaction of neighboring particles via the fluid phase, while the second is the result of direct collision of particle. Interaction of the first type may be expected to produce a comparatively slow and smooth change of particle velocity $\mathbf{W}$, while collisions are characterized by abrupt variations of particle velocity. Direct collisions of particles evidently predominate in the limit case of concentrated systems of very large and heavy particles suspended in a low viscosity gas stream. Such processes were considered in [4]. In the majority of real systems collisions play a secondary role, and velocity $\mathbf{W}$ changes in the main under the influence of the first type of interaction forces. This conclusion was reached, for example, in [5] on the basis of experiments in which a close analogy was observed between small scale pulsations of particles in a packet and Brownian motions, as expressed by the proportionality of translation in a very small time interval to the root of that interval magnitude. The low collision frequency observed in [5] may possible be explained by an attenuation of the relative velocity of particles in the process of their convergence, and the squeezing out of the gas film, so that even if abrupt velocity changed did occur, their effect was comparatively insignificant.

In accordance with the theory of random processes of the type of Brownian movements [7] we may expect that there exists such a time interval Δt during which velocity $\mathbf{W}$ remains practically unchanged, while force $\mathbf{F}_1$ undergoes a considerable number of fluctuations. In that case with $\mathbf{F}_2 \ll \mathbf{F}_1$ the Fokker-Planck equation is valid for the particle distribution function with respect to small scale pulsation velocities. If $\mathbf{F}_2 \gtrsim \mathbf{F}_1$ and the velocity of a particle does not substantially change in the course of its free run (in this connection see arguments in [4]), then this equation must be supplemented by the collision term. On the assumption of spatial homogeneity we obtain [7]

$$\frac{\partial f}{\partial t} = \frac{\alpha}{m}\frac{\partial (fW_i)}{\partial W_i} + \frac{B}{m}\frac{\partial^2 f}{\partial W_i \partial W_i} + C(ff_1) \qquad \left(B = \frac{\alpha\theta}{m}\right) \tag{5.2}$$

Here $C(ff_1)$ is the collision integral in its conventional form, θ the effective temperature of small scale motion ($\theta = \frac{1}{3}\, m \langle W^2 \rangle$) and B the coefficient of diffusion in the domain of individual particle velocities. We note that Eq. (5.2) is related to the small scale motion of a particle only, and not to its complete motion within a packet, contrary to the situation considered in [11 and 12].

The collision term of (5.2) may evidently be neglected for small $\mathbf{W}$ and not too great ρ. We then have a complete analogy with the Brownian motion as in the experiments of [5]. In this case the relation between D and θ is expressed by

$$D = \theta / \alpha \tag{5.3}$$

and the stationary distribution function f is of the Maxwellian form, as it was in another limit case considered in [4].

By virtue of (4.1) and (5.3) the complete mean square velocity of a particle is represented to within the accuracy of the diffusion coefficient D by an expression of the form

$$\langle \delta w^2 \rangle = \frac{\rho^2(\rho_* - \rho)}{\rho_*}\left[\left(\frac{2}{1-\rho} + \frac{d\ln K}{d\rho}\right)^2 u^2 + \frac{K}{(1-\rho)^3}\frac{\beta D}{d_2}\right] \times$$
$$\times \left[1 - \left(1 + \frac{a}{h}\right)\exp\left(-\frac{a}{h}\right)\right] + \frac{3\alpha D}{m} \tag{5.4}$$

Parameter D, and therefore also θ may in principle be determined by equating the energy flux E_1 induced by the large scale motion to that of small scale motion energy dissipation

E_2 of the latter in the form of heat [4]. A detailed analysis of energy phenomena in the stream is outside the scope of this paper. We shall however note that if the viscous dissipation of energy of local perturbations of the carrying stream is neglected, then the flow of energy E_1 can obviously be considered equal to the energy dissipation of large scale motions in consequence of the irreversible process of momentum transfer by small scale isotropic motions. For the latter the conventional in viscous fluid hydrodynamics relationship with the effective dynamic viscosity $\rho d_2 D$ is valid. From (2.5), (2.6), (3.3) and (3.7) we obtain after computations

$$E_1 = \rho d_2 D \int d\omega \int dk k^2 f_{wtwt} = \frac{\rho^2(\rho_* - \rho)}{\rho_*} \beta K \left[\left(\frac{2}{1-\rho} + \right.\right.$$

$$\left.\left. + \frac{d \ln K}{d\rho} \right)^2 \frac{u^2}{1-\rho} + \frac{K}{(1-\rho)^4} \frac{\beta D}{d_2} \right] \left(1 + \frac{a}{h}\right) \exp\left(-\frac{a}{h}\right) \tag{5.5}$$

The small scale motion energy dissipation is evidently equal to

$$E_2 = n\alpha \langle W^2 \rangle = \frac{3\rho\alpha^2 !}{d_2 (v^\circ)^2} D \tag{5.6}$$

Equating Expressions (5.5) and (5.6), and using expressions of u and β from (1.1) and (1.2), we obtain for D the following Eq.:

$$\left[3 - \frac{\rho^2(\rho_* - \rho)}{\rho_*} \left(\frac{K}{S}\right)^2 \frac{1}{(1-\rho)^4} \left(1 + \frac{a}{h}\right) \exp\left(-\frac{a}{h}\right) \right] D =$$

$$= \frac{8}{729} \frac{\rho^2(\rho_* - \rho)}{\rho_*} \frac{1-\rho}{KS^2} \left(\frac{2}{1-\rho} + \frac{d \ln K}{d\rho} \right)^2 \left(1 + \frac{a}{h}\right) \exp\left(-\frac{a}{h}\right) D_0 \tag{5.7}$$

$$D_0 = g^2 (d_2 a^2 / \mu_0)^3$$

Parameter h in (5.7) was defined in (4.1), and $S(\rho)$ is a function of the effective viscosity μ of the gas flowing past the ordered lattice of particles, $\mu = \mu_0 S(\rho)$, $S(0) = 1$, $dS/d\rho > 0$ (see [6]).

It is not difficult to see that Eq. (5.7) has a finite root D, only if parameter ρ differs considerably from zero, or ρ_*, and parameter D_0 is sufficiently great. If these conditions are not fulfilled, then Eq. (5.7) has a single root $D = 0$ only, i.e., small scale motions are absent. In this case there exist in the system only vertical pseudoturbulent pulsations of intensity

$$\langle w'^2 \rangle = \frac{\rho^2(\rho_* - \rho)}{\rho_*} \left(\frac{2}{1-\rho} + \frac{d \ln K}{d\rho} \right)^2 u^2 \tag{5.8}$$

This last conclusion is in a qualitative agreement with the results of numerous experiments which prove that with increasing μ_0 and decreasing ρ, d_2, a the oscillation of particles in a packet ceases.

As shown by the expressions here derived the velocities of random motion are by no means negligible, even at small values of ρ, or $\rho_* - \rho$. Hence, these results are applicable to highly rarefied systems only. However the method used here may be applied also to the general case by introducing into the convection terms of equations of Section 1 of corrections derived in Section 4, and complementing Eq. (1.5) of the dispersed phase by that phase viscosity and pressure terms associated with the small scale motions of particles. The accomplishment of such a program, and the investigation of the various characteristics of random motions in terms of the averaged motions and of the physical parameters of the phases is, of course, a problem of its own.

BIBLIOGRAPHY

1. Barenblatt, G.I., On the motion of particles suspended in a turbulent stream. PMM, Vol. 17, No. 3, 1953.
2. Rakhmatulin, Kh.A., Fundamental gas dynamics of interpenetrating motions of compressible media. PMM, Vol. 20, No. 2, 1956.
3. Murray, J.D., On the mathematics of fluidization. Pt. 1, Fundamental equations and wave propagation. J. Fluid Mech., Vol. 21, Pt. 3, 1965.
4. Buevich, Iu.A., The approximate theory of the suspended layer. PMTF, No. 6, 1966.
5. Todes, O.M., Bondareva, A.K. and Grinbaum, M.B., The motion and the mixing of solid phase particles in a fluidized layer. Khim. Prom., No. 6, 1966.
6. Buevich, Iu.A. and Safrai, V.M., Fluid phase viscosity in disperse systems. PMTF, No. 6, 1967.
7. Chandrasekhar, S., Stochastic Problems of Physics and Astronomy. (Russian transl.) Izd. Inostr. Lit., 1947.
8. Iaglom, A.M., Introduction to the theory of stationary random functions. Usp. Mat. Nauk, Vol. 7, No. 5, 1952.
9. Buevich, Iu.A. and Leonov, A.I., Filtration of a fluid through a random porosity medium Izv. Akad. NaukUSSR, MZhG, No. 6, 1967.
10. Münster, A., Theory of fluctuations. Termodinamica del processi irreversibili. Bologna, 1960.
11. Houghton, G., Particle and fluid diffusion in homogeneous fluidization. Industr. Engrng. Fundamentals, Vol. 5, No. 2, 1966.
12. Miasnikov, V.P., On the dynamic equations of two-component systems. PMTF, No. 2, 1967.
13. Monin, A.S. and Iaglom, A.M., Statistical Hydromechanics. Mechanics of Turbulence. Pt. 1. M. "Nauka", 1965.

Translated by J.J.D.